For the function $f(x) = x^2 - 6x$ on the interval $x \in [-2, 4]$, find the value of c such that $f_{ave} = f(c)$. **SCORE: _____ / 7 PTS**

NOTE: This is the value c guaranteed by the Mean Value Theorem for Integrals.

$$f_{AVE} = \boxed{\frac{\int_{-2}^{4} (x^2 - 6x)\,dx}{4 - -2}} = \frac{1}{6}\left(\boxed{\frac{1}{3}x^3 - 3x^2}\right)\Big|_{-2}^{4} = \frac{1}{18}(64 - -8) - \frac{1}{2}(16 - 4)$$

2 1

$$= \frac{1}{18}72 - \frac{1}{2}\cdot 12$$

$$= 4 - 6 = \boxed{-2}$$

1

$$\boxed{c^2 - 6c = -2}$$ 1

$$c^2 - 6c + 2 = 0$$

$$c = \frac{6 \pm \sqrt{36 - 8}}{2} = \frac{6 \pm \sqrt{28}}{2} = \frac{6 \pm 2\sqrt{7}}{2} = \boxed{3 \pm \sqrt{7}}$$ 1

$$c = \boxed{3 - \sqrt{7}} \in (-2, 4)$$

1

Find the surface area if the curve $x = \frac{1}{3}(y^2+2)^{\frac{3}{2}}$ for $y \in [1, 2]$ is revolved around the $x-$axis. **SCORE: _____ / 7 PTS**

$\underbrace{\quad}_{g(y)}$ $r = y$

$$g'(y) = \frac{1}{3} \cdot \frac{3}{2}(y^2+2)^{\frac{1}{2}} \cdot 2y = y\sqrt{y^2+2}$$

$$2\pi \int_1^2 y\sqrt{1 + (y\sqrt{y^2+2})^2}\, dy \quad ② \quad ②$$

$$= 2\pi \int_1^2 y\sqrt{1 + y^2(y^2+2)}\, dy$$

$$= 2\pi \int_1^2 y\sqrt{1 + 2y^2 + y^4}\, dy$$

$$= 2\pi \int_1^2 y(1+y^2)\, dy \quad ①$$

$$= 2\pi \int_1^2 (y+y^3)\, dy$$

$$① \quad = 2\pi \left(\frac{1}{2}y^2 + \frac{1}{4}y^4\right)\Big|_1^2$$

$$= \pi\left(y^2 + \frac{1}{2}y^4\right)\Big|_1^2$$

$$= \pi\left((4-1) + \frac{1}{2}(16-1)\right)$$

$$= \pi\left(3 + \frac{15}{2}\right)$$

$$= \frac{21\pi}{2}$$

①

A company which manufactures plant-based foods conducted a focus group to test the public response to their latest synthetic meat product. The members of a focus group were each given a 4 ounce portion of the synthetic meat. Members were then randomly selected from the group, and X is the random variable representing the amount of the meat product the member ate (measured in ounces). Find the mean (average) amount of food eaten per member if the probability density function is given by

SCORE: _____ / 8 PTS

$$f(x) = \begin{cases} k\sqrt{16-x^2}, & x \in [0,4] \\ 0, & x \notin [0,4] \end{cases}$$ (for some appropriate constant k).

$$\int_0^4 k\sqrt{16-x^2}\,dx = 1 \quad 1$$

$$k\int_0^4 \sqrt{16-x^2}\,dx = 1$$

$$k \cdot \tfrac{1}{4}\pi 4^2 = 1 \quad \tfrac{1}{2}$$

$$k = \frac{1}{4\pi} \quad \tfrac{1}{2}$$

$$\int_0^4 \frac{1}{4\pi} x\sqrt{16-x^2}\,dx \quad 2$$

$$= \frac{1}{4\pi}\int_{16}^{0} -\tfrac{1}{2}\sqrt{u}\,du \quad 2$$

$$= -\frac{1}{8\pi}\,\frac{2}{3}u^{\frac{3}{2}}\Big|_{16}^{0} \quad 1$$

$$= -\frac{1}{12\pi}\left(-16^{\frac{3}{2}}\right)$$

$$= \frac{1}{12\pi}\cdot 64$$

$$= \frac{16}{3\pi} \quad 1$$

$u = 16 - x^2$

$du = -2x\,dx$

$-\tfrac{1}{2}du = x\,dx$

$x = 0 \rightarrow u = 16$

$x = 4 \rightarrow u = 0$

Find the arclength function for the curve $y = \arcsin x + \sqrt{1-x^2}$ with starting point $x = 0$. **SCORE: _____ / 8 PTS**

$$f'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{1-x^2}} \cdot -2x = \frac{1-x}{\sqrt{1-x^2}}$$

$$\boxed{\int_0^x \sqrt{1 + \left(\boxed{\frac{1-t}{\sqrt{1-t^2}}}\right)^2}\, dt} \quad 2 \quad 2$$

$$= \int_0^x \sqrt{1 + \frac{1-2t+t^2}{1-t^2}}\, dt$$

$$= \int_0^x \sqrt{\frac{1-t^2+1-2t+t^2}{1-t^2}}\, dt$$

$$= \int_0^x \sqrt{\frac{2-2t}{1-t^2}}\, dt$$

$$= \sqrt{2} \int_0^x \sqrt{\frac{1-t}{1-t^2}}\, dt$$

$$= \boxed{\sqrt{2} \int_0^x \frac{1}{\sqrt{1+t}}\, dt} \quad 2 \qquad u = 1+t$$

$$= \sqrt{2} \cdot \boxed{2\sqrt{1+t}}\Big|_0^x \quad 1$$

$$= \boxed{2\sqrt{2}\left(\sqrt{1+x} - 1\right)} \quad 1$$